

# Syllabus<sup>1</sup>

## 1 Introduction

1.1 This syllabus lists topics which may be used for the International Olympiad of Metropolises.

### 1.2 Character of the problems

Problems should focus on testing creativity and understanding of physics rather than testing mathematical virtuosity or speed of working. The proportion of marks allocated for mathematical manipulations should be kept small. In the case of mathematically challenging tasks, alternative approximate solutions should receive partial credit. Problem texts should be concise; the theoretical and the experimental examination texts should each contain fewer than 12000 characters (including white spaces, but excluding cover sheets and answer sheets).

### 1.3 Exceptions

Questions may contain concepts and phenomena not mentioned in the Syllabus providing that sufficient information is given in the problem text so that students without previous knowledge of these topics would not be at a noticeable disadvantage. Such new concepts must be closely related to the topics included in the syllabus. Such new concepts should be explained in terms of topics in the Syllabus.

### 1.4 Units

Numerical values are to be given using SI units, or units officially accepted for use with the SI. It is assumed that the contestants are familiar with the phenomena, concepts, and methods listed below, and are able to apply their knowledge creatively.

## 2 Theoretical skills

### 2.1 General

The ability to make appropriate approximations, while modelling real life problems.

### 2.2 Mechanics

#### 2.2.1 Kinematics

Velocity and acceleration of a point particle as the derivatives of its displacement vector. Linear speed; centripetal and tangential acceleration. Motion of a point particle with a constant acceleration. Addition of velocities and angular velocities; addition of accelerations without the Coriolis term; recognition of the cases when the Coriolis acceleration is zero. Motion of a rigid body as a rotation around an instantaneous centre of rotation; velocities and accelerations of the material points of rigid rotating bodies.

#### 2.2.2 Statics

Finding the centre of mass of a system via summation or via integration. Equilibrium conditions: force balance (vectorially or in terms of projections), and torque balance (only for one- and two-dimensional geometry). Normal force, tension force, static and kinetic friction force; Hooke's law, stress, strain, and Young modulus. Stable and unstable equilibria.

#### 2.2.3 Dynamics

Newton's second law (in vector form and via projections (components)); kinetic energy for translational and rotational motions. Potential energy for simple force fields (also as a line integral of the force field). Momentum, angular momentum, energy and their conservation laws. Mechanical work and power; dissipation due to friction. Inertial and non-inertial frames of reference: inertial force, centrifugal force,

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<sup>1</sup> The Syllabus of International Olympiad of Metropolises based on the materials from IPhO Syllabus <http://ipho.org/syllabus.html>

potential energy in a rotating frame. Moment of inertia for simple bodies (ring, disk, sphere, hollow sphere, rod), parallel axis theorem; finding a moment of inertia via integration.

#### 2.2.4 Celestial mechanics

Law of gravity, gravitational potential, Kepler's laws (no derivation needed for first and third law). Energy of a point mass on an elliptical orbit.

#### 2.2.5 Hydrodynamics

Pressure, buoyancy, continuity law, the Bernoulli equation. Surface tension and the associated energy, capillary pressure;

### 2.3 Electromagnetic fields

#### 2.3.1 Basic concepts

Concepts of charge and current; charge conservation and Kirchhoff's current law. Coulomb force; electrostatic field as a potential field; Kirchhoff's voltage law. Magnetic  $B$ -field; Lorentz force; Ampère's force; Biot-Savart law and  $B$ -field on the axis of a circular current loop and for simple symmetric systems like straight wire, circular loop and long solenoid.

#### 2.3.2 Integral forms of Maxwell's equations

Gauss' law (for  $E$ - and  $B$ -fields); Ampère's law; Faraday's law; using these laws for the calculation of fields when the integrand is almost piece-wise constant. Boundary conditions for the electric field (or electrostatic potential) at the surface of conductors and at infinity; concept of grounded conductors. Superposition principle for electric and magnetic fields; uniqueness of solution to wellposed problems; method of image charges.

#### 2.3.3 Interaction of matter with electric and magnetic

Fields Resistivity and conductivity; differential form of Ohm's law. Dielectric and magnetic permeability; relative permittivity and permeability of electric and magnetic materials; energy density of electric and magnetic fields; ferromagnetic materials; hysteresis and dissipation; eddy currents; Lenz's law. Surface charge due to dielectric polarisation (qualitatively); surface current due to magnetization (qualitatively); continuity conditions for fields at the surface of dielectric and ferromagnetic materials. Charges in magnetic field: helicoidal motion, cyclotron frequency, drift in crossed  $E$ - and  $B$ -fields. Energy of a magnetic dipole in a magnetic field; dipole moment of a current loop.

#### 2.3.4 Circuits

Linear resistors and Ohm's law; Joule's law; work done by an electromotive force; ideal and non-ideal batteries, constant current sources, ammeters, voltmeters and ohmmeters. Nonlinear elements of given  $V$ - $I$  characteristic. Capacitors and capacitance (also for a single electrode with respect to infinity); self-induction and inductance; energy of capacitors and inductors; mutual inductance; transformers with closed ferromagnetic cores; time constants for  $RL$  and  $RC$  circuits. AC circuits: complex amplitude; impedance of resistors, inductors, capacitors, and combination circuits; phasor diagrams; current and voltage resonance; active power.

### 2.4 Oscillations and waves

#### 2.4.1 Single oscillator

Harmonic oscillations : equation of motion, frequency, angular frequency and period. Physical pendulum and its reduced length. Behaviour near unstable equilibria. Exponential decay of damped oscillations; resonance of sinusoidally forced oscillators: amplitude and phase shift of steady state oscillations. Free oscillations of  $LC$ - circuits; mechano-electrical analogy; positive feedback as a source of instability; generation of sine waves by feedback in a  $LC$ -resonator.

### 2.4.2 Coupled oscillators

Coupled harmonic oscillators of many degrees of freedom: equation of motion, natural frequencies and natural modes, physical interpretation of zero frequencies, arbitrary oscillations as superpositions of natural modes.

### 2.4.3 Waves

Propagation of harmonic waves: phase as a linear function of space and time; wave length, wave vector, phase and group velocities; exponential decay for waves propagating in dissipative media; transverse and longitudinal waves; the classical Doppler effect. Waves in inhomogeneous media: Fermat's principle, Snell's law. Sound waves: speed as a function of pressure (Young's or bulk modulus) and density, Mach cone. Wave speed in strings and for gravity waves in shallow water. Energy carried by waves: proportionality to the square of the amplitude, continuity of the energy flux.

### 2.4.4 Interference and diffraction

Superposition of waves: coherence, beats, standing waves, Huygens' principle (integral expression of the amplitude only for small propagation angles), interference due to thin films (conditions for intensity minima and maxima only). Diffraction from one and two slits, diffraction grating, Bragg reflection.

### 2.4.5 Interaction of electromagnetic waves with matter

Dependence of electric permittivity on frequency (qualitatively); refractive index; dispersion and dissipation of electromagnetic waves in transparent and opaque materials. Linear polarisation; Brewster angle; polarisers; Malus' law. Circular and elliptic polarisation as a superposition of linearly polarised waves. Double refraction (only for axial propagation), quarter-wave plate, circular polarisers. Rotation of polarisation plane in optically active media.

### 2.4.6 Geometrical optics and photometry

Approximation of geometrical optics: rays and optical images; a partial shadow and full shadow. Thin lens approximation; construction of images created by ideal thin lenses; thin lens equation. Luminous flux and its continuity; illuminance; luminous intensity.

### 2.4.7 Optical devices

Telescopes and microscopes: magnification and resolving power; diffraction grating and its resolving power; interferometers.

## 2.5 Relativity

Principle of relativity and Lorentz transformations for the time and spatial coordinate, and for the energy and momentum; mass-energy equivalence; invariance of the spacetime interval and of the rest mass. Addition of parallel velocities; time dilation; length contraction; relativity of simultaneity; energy and momentum of photons and relativistic Doppler effect; relativistic equation of motion; conservation of energy and momentum for elastic and non-elastic interaction of particles.

## 2.6 Quantum Physics

### 2.6.1 Probability waves

Particles as waves: relationship between the frequency and energy, and between the wave vector and momentum. Wave function as a probability wave; energy levels of hydrogen-like atoms (circular orbits only) and of parabolic potentials; quantization of angular momentum. Uncertainty principle for the conjugate pairs of time and energy, and of coordinate and momentum (as a theorem, and as a tool for estimates);

### 2.6.2 Structure of matter

Emission and absorption spectra for hydrogen-like atoms (for other atoms — qualitatively), and for molecules due to molecular oscillations; spectral width and lifetime of excited states. Pauli exclusion principle for Fermi particles. Particles (knowledge of charge and spin): electrons, electron neutrinos, protons, neutrons, photons; Compton scattering. Protons and neutrons as compound particles. Atomic nuclei, energy levels of nuclei (qualitatively); alpha-, beta- and gamma-decays; fission, fusion and neutron capture; mass defect; half life and exponential decay. Crystal structures: crystal planes (relevance to Bragg reflection), electronic energy levels (qualitatively, metals versus dielectric materials and semiconductors); photoelectric effect.

## 2.7 Thermodynamics and statistical physics

### 2.7.1 Classical thermodynamics

Concepts of thermal equilibrium and reversible processes; internal energy, work and heat; Kelvin's temperature scale; entropy; open, closed, isolated systems; first and second laws of thermodynamics. Kinetic theory of ideal gases: Avogadro number, Boltzmann factor and gas constant; translational motion of molecules and pressure; ideal gas law; translational, rotational and oscillatory degrees of freedom; equipartition theorem; internal energy of ideal gases; root-mean-square speed of molecules. Isothermal, isobaric, isochoric, and adiabatic processes; specific heat for isobaric and isochoric processes; forward and reverse Carnot cycle on ideal gas and its efficiency; efficiency of non-ideal heat engines.

### 2.7.2 Heat transfer and phase transitions

Phase transitions (boiling, evaporation, melting, sublimation) and latent heat; saturated vapour pressure, relative humidity; boiling; Dalton's law; concept of heat conductivity; continuity of heat flux.

### 2.7.3 Statistical physics

Planck's law (explained qualitatively, does not need to be remembered), Wien's displacement law; the Stefan-Boltzmann law.

## 3 Experimental skills

### 3.1 Introduction

The theoretical knowledge required for carrying out the experiments must be covered by Section 2 of this Syllabus.

The experimental problems should contain at least some tasks for which the experimental procedure (setup, the list of all the quantities subject to direct measurements, and formulae to be used for calculations) is not described in full detail.

The experimental problems may contain implicit theoretical tasks (deriving formulae necessary for calculations); there should be no explicit theoretical tasks unless these tasks test the understanding of the operation principles of the given experimental setup or of the physics of the phenomena to be studied, and do not involve long mathematical calculations.

The expected number of direct measurements and the volume of numerical calculations should not be so large as to consume a major part of the allotted time: the exam should test experimental creativity, rather than the speed with which the students can perform technical tasks.

The students should have the following skills.

### 3.2 Safety

Knowing standard safety rules in laboratory work. Nevertheless, if the experimental set-up contains any safety hazards, the appropriate warnings should be included in the text of the problem. Experiments with major safety hazards should be avoided.

### 3.3 Measurement techniques and apparatus

Being familiar with the most common experimental techniques for measuring physical quantities mentioned in the theoretical part.

Knowing commonly used simple laboratory instruments and digital and analog versions of simple devices, such as calipers, the Vernier scale, stopwatches, thermometers, multimeters (including ohmmeters and AC/DC voltmeters and ammeters), potentiometers, diodes, , lenses, prisms, optical stands, calorimeters, and so on.

Sophisticated practical equipment likely to be unfamiliar to the students should not dominate a problem. In the case of moderately sophisticated equipment (such as oscilloscopes, counters, ratemeters, signal and function generators, photogates, etc), instructions must be given to the students.

### 3.4 Accuracy

Being aware that instruments may affect the outcome of experiments.

Being familiar with basic techniques for increasing experimental accuracy (e.g. measuring many periods instead of a single one, minimizing the influence of noise, etc).

Knowing that if a functional dependence of a physical quantity is to be determined, the density of taken data points should correspond to the local characteristic scale of that functional dependence.

Expressing the final results and experimental uncertainties with a reasonable number of significant digits, and rounding off correctly.

### 3.5 Experimental uncertainty analysis

Identification of dominant error sources, and reasonable estimation of the magnitudes of the experimental uncertainties of direct measurements (using rules from documentation, if provided).

Distinguishing between random and systematic errors; being able to estimate and reduce the former via repeated measurements.

Finding absolute and relative uncertainties of a quantity determined as a function of measured quantities using any reasonable method (such as linear approximation, addition by modulus or Pythagorean addition).

### 3.6 Data analysis

Transformation of a dependence to a linear form by appropriate choice of variables and fitting a straight line to experimental points. Finding the linear regression parameters (gradient, intercept and uncertainty estimate) either graphically, or using the statistical functions of a calculator (either method acceptable).

Selecting optimal scales for graphs and plotting data points with error bars.

## 4 Mathematics

### 4.1 Algebra

Simplification of formulae by factorisation and expansion. Solving linear systems of equations. Solving equations and systems of equations leading to quadratic and biquadratic equations; selection of physically meaningful solutions. Summation of arithmetic and geometric series.

### 4.2 Functions

Basic properties of trigonometric, inverse-trigonometric, exponential and logarithmic functions and polynomials. Solving simple equations involving trigonometric, inverse-trigonometric, logarithmic and exponential functions.

### 4.3 Geometry and stereometry

Degrees and radians as alternative measures of angles. Equality of alternate interior and exterior angles, equality of corresponding angles. Recognition of similar triangles. Areas of triangles, trapezoids, circles and ellipses; surface areas of spheres, cylinders and cones; volumes of spheres, cones, cylinders and prisms. Sine and cosine rules, property of inscribed and central angles, Thales' theorem.

### 4.4 Vectors

Basic properties of vectorial sums, dot and cross products. Double cross product and scalar triple product. Geometrical interpretation of a time derivative of a vector quantity.

#### 4.5 Statistics

Calculation of probabilities as the ratio of the number of objects or event occurrence frequencies. Calculation of mean values, standard deviations, and standard deviation of group means.

#### 4.6 Calculus

Finding derivatives of elementary functions, their sums, products, quotients, and nested functions. Integration as the inverse procedure to differentiation. Finding definite and indefinite integrals in simple cases: elementary functions, sums of functions, and using the substitution rule for a linearly dependent argument. Making definite integrals dimensionless by substitution. Geometric interpretation of derivatives and integrals. Finding constants of integration using initial conditions. Concept of gradient vectors (partial derivative formalism is not needed).

#### 4.7 Approximate and numerical methods

Using linear and polynomial approximations based on Taylor series. Linearization of equations and expressions. Perturbation method: calculation of corrections based on unperturbed solutions. Numerical integration using the trapezoidal rule or adding rectangles.